Jayne Overgard
Math 629
HW6

**Carl Friedrich Gauss (1777 – 1855)**

Carl Friedrich Gauss, born in Braunschweig, Germany on April 30, 1777, is known as a mathematician, a physicist, and an astronomer. He was born to a poor and relatively uneducated family, but his mathematical abilities were noticed very early on, even as a toddler, and he was subsequently mentored and encouraged by an uncle and his teachers. His father had actively discouraged his studies, wishing him to become a common laborer much like himself, but his mother and in particular this intellectually curious uncle (his mother's brother), actively encouraged his studies and were able to successfully thwart his father's plans.

In his earliest years Gauss focused on number theory and geometry. One of his most memorable times was at age nine in school when his instructor asked the class to add up 100 numbers of an arithmetic sequence. Gauss was able to complete the feat immediately, while the other students worked on it most of the period. Gauss was the only one who got the correct answer, and by coincidence in his class was the teacher's seventeen year old helper, Martin Bartels, who himself had quite an interest in mathematics. Bartels normally helped students with writing and sharpened their quill pens, but he actively encouraged Gauss, in whom he recognized great potential.

Through Bartels' connections Gauss was able to secure funding from the Duke of Braunschweig when he was 14 years old. Gauss was then able to devote his time to his studies for many years due to this money; first he went to the Collegium Carolinum in Braunschweig for three years, then went on to the University at Göttingen, Germany

where he studied mathematics. Also, Bartels continued to mentor Gauss for some years. Bartels himself became a famous mathematician, and he and Gauss maintained a life-long friendship.

Gauss, before the age of twenty, had been able to prove that a heptadecagon could be constructed using a compass and a straightedge (Weisstein, 2007). He published this and much more in his famous work called "*Disquisitiones Arithmeticae*" in 1801. This book included number theory based on integers, and a presentation of the idea that the integers *b* and *c* are congruent relative to the modulus *a* if *a* divides the difference of *b* and *c*. (Katz, 2004). Also in the book is a proof of the law of quadratic reciprocity. This law describes when two odd primes will be residues of each other.

In order to attempt to prove the law of quartic reciprocity, Gauss studied what are now referred to as "Gaussian integers", or complex numbers. The idea here is that since an odd prime can be expressed as $p = a^2 + b^2$ if and only if it is of the form $4n+1,$ then these primes are composite: $p = (a+bi)(a-bi)$. (Katz, 2004).

Gauss also worked on cyclotomic equations, those of the form $x^n - 1 = 0$. He concentrated his work on $x^{n-1} + x^{n-2} + ... + x + 1 = 0$; he only worked with *n*'s that were prime. His doctoral thesis was a proof of the Fundamental Theorem of Algebra, that every polynomial equation with complex coefficients has at least one root.

Gaussian elimination, a method used to solve systems of linear equations, and the Gauss-Jordan method, also having to do with solving linear algebra problems, were developed as a response to using Cramer's rule for more difficult linear equations with rational coefficients. Gauss also developed and improved upon Legendre's work on least squares. Gauss used Legendre's least squares method in his *Theoria motus corporum*

*coelestium in sectionibus conicis solem ambientum (Theory of Motion of the Heavenly Bodies),* published in 1809, but did not give Legendre credit, claiming he had developed the idea himself in 1795. The astronomy book calculated the planetary orbits and proved the formulas for the orbits. Although Legendre was put out by the use of what he had himself published in 1805, Gauss did go further with the method of least squares and used his Gaussian elimination method in the process, and also developed much in the direction of statistics, including the idea of normal distribution.

After Gauss published his famous 1801 work, one of the many letters of acknowledgement he received was from a Mr. Leblanc who offered both praise and suggestions. Mr. Leblanc was a bit of a mathematician himself, and Gauss was impressed by his comments and observations. After much correspondence Mr. Leblanc revealed that he was actually Sophie Germain, but had worried that Gauss would not take her seriously as a woman. She was forced to admit her true identity when she intervened with the French government on Gauss' behalf. Of this revelation Gauss wrote,

> "*But how describe to you my admiration and astonishment at seeing my esteemed correspondent Mr. Leblanc metamorphose himself into this illustrious personage [Sophie Germain] who gives such a brilliant example of what I would find it difficult to believe. A taste for the abstract sciences in general and above all the mysteries of numbers is excessively rare: one is not astonished at it; the enchanting charms of this sublime science reveal themselves only to those who have the courage to go deeply into it. But when a person of the sex which, according to our customs and prejudices, must encounter infinitely more*

*difficulties than men to familiarize herself with these thorny researches, succeeds nevertheless in surmounting these obstacles and penetrating the most obscure parts of them, then without doubt she must have the noblest courage, quite extraordinary talents and a superior genius.*" (Newman, 1956).

Also in 1801 a small dwarf planet Ceres was discovered and watched for only a few days before disappearing behind the sun. With such little data the astronomer Piazzi who discovered the planet was unable to predict where and when the planet would reappear. Gauss solved the problem and corrected predicted its reappearance within one half of one degree.

Gauss' funding ended when the Duke was killed in a war against Napoleon. Gauss was 30 at the time. After the Duke's death Gauss was in need of a job, and while there were job offerings in Russia, Germany was eager to hold on to the now famous mathematician. In 1807 he became the director of the observatory at Göttingen, Germany. There he worked in the observatory but also taught mathematics at the University at Göttingen as part of his employment. He lived there until he died at age 77 on February 23, 1855.

While at the university Gauss had some quite talented students studying under his wing, including Friedrich Bessel, Christoph Gudemann, J. W. Richard Dedekind, Johann Encke, Johann Listing, and Bernhard Riemann (Wikipedia, 2007).

Gauss also was an inventor. In 1818 he invented the heliotrope, a device used in land surveying.

On a personal level Gauss was not entirely financially secure in adulthood, married twice, and had six children. His first wife, the love of his life, died shortly after the birth of their third child. His first son had some mathematical ability, but it appears he may not have gotten along well with his other sons and two of them immigrated to the United States. Although he corresponded extensively with other mathematicians, he had very few nearby friends and worked alone most of the time. He was quite insecure about his work, always the perfectionist, and so did not publish most of his findings. Many proofs were discovered later, in a personal journal he started when he proved the heptodecagon findings.

**References**

Katz, Victor J. (2004). *A History of Mathematics*, p. 386, 401-419, 452, 456-457, 463-464.

Newman, James R., (1956). *The World of Mathematics,* Volume I, p. 333.

Weisstein, Eric. (2007), *World of Biography*, http://scienceworld.wolfram.com/biography/Gauss.html.

Wikipedia, (2007), http://en.wikipedia.org/wiki/Carl_Friedrich_Gauss