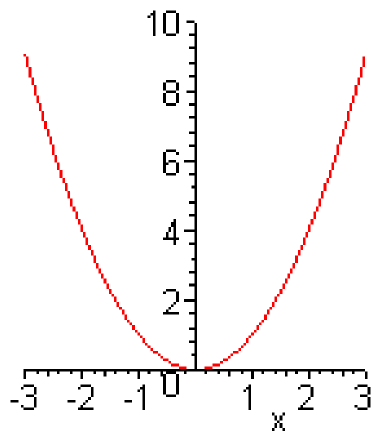
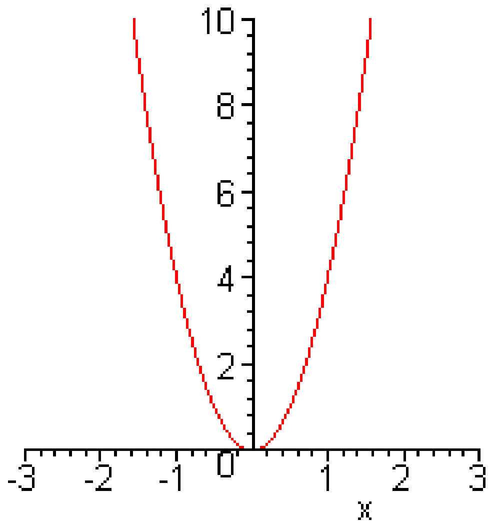
Analyzing Quadratic Equations
A Tutorial for Algebra Students

The purpose of this tutorial is to familiarize the student with how subtle changes to a quadratic equation can affect the graph of the equation, and when solving quadratic equations, how the roots tie in with the value of the **discriminant**.

- First, let us examine a very simple example of a quadratic, namely $\mathbf{y = x^2}$. The graph of this equation looks like this:

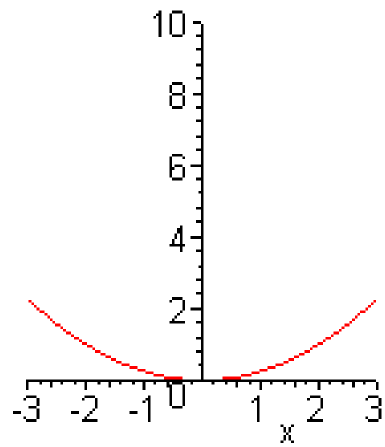


- Now, what happens when we alter the coefficient on the $\mathbf{x^2}$ term? For our two examples we will graph $\mathbf{y = 4x^2}$ and $\mathbf{y = (1/4)x^2}$.



$\mathbf{y = 4x^2}$

Here, the coefficient is greater than one. Note that the parabola is narrower.

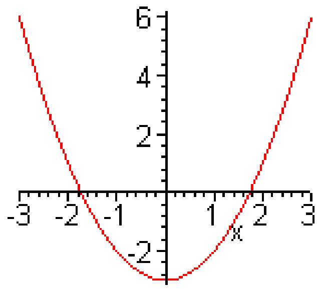


$\mathbf{y = (1/4)x^2}$

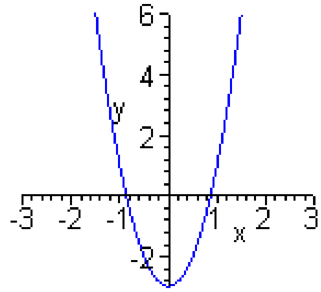
Here the coefficient is between zero and one. Note that the parabola is wider.

- What happens when we make the coefficients on the $\mathbf{x^2}$ term negative? Here are examples of three graphs with positive coefficients, and underneath are graphs with negative coefficients:

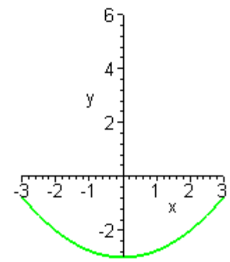
$y = x^2 - 3$

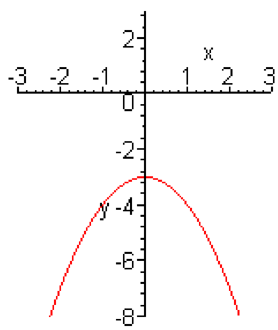


$y = 4x^2 - 3$

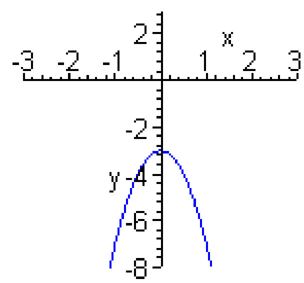


$y = (1/4)\, x^2 - 3$

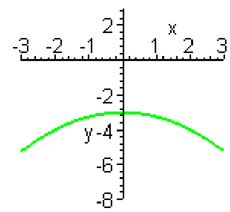




$y = -x^2 – 3$



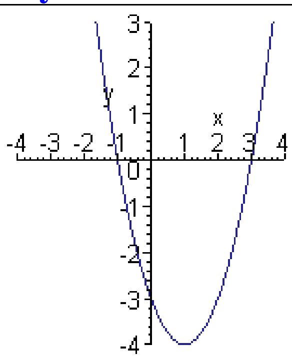
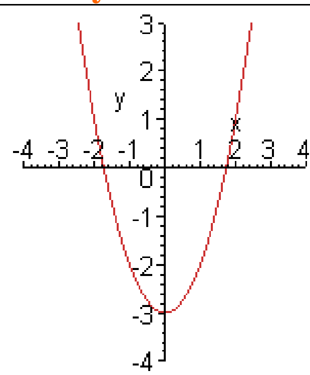
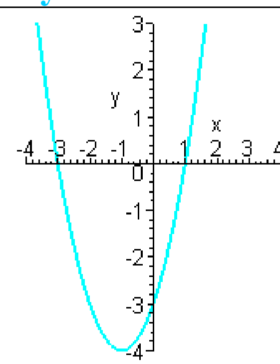
$y = -4x^2 – 3$



$y = -(1/4)\, x^2 – 3$

Note that changing the coefficient on the $\mathbf{x^2}$ term to a negative flips the parabola. One more thing to note here: a constant term of **–3** has been added to each of these examples to demonstrate that the vertex for each parabola, including the inverted ones, is at **(0, –3).** This is because when the x term is zeroed out, y = -3.

- Inclusion of an x term shifts the graph horizontally, to the left or right. This is left for the student to explore. Three examples are shown below:

$\mathbf{y = x^2 – 2x – 3}$	$\mathbf{y = x^2 – 3}$	$\mathbf{y = x^2 + 2x – 3}$
		

*Interesting note: the middle term causes the vertex to adjust as well.

The Discriminant in Solving Quadratic Equations

Given an equation in the form

$$a\,x^2 + b\,x + c = 0$$

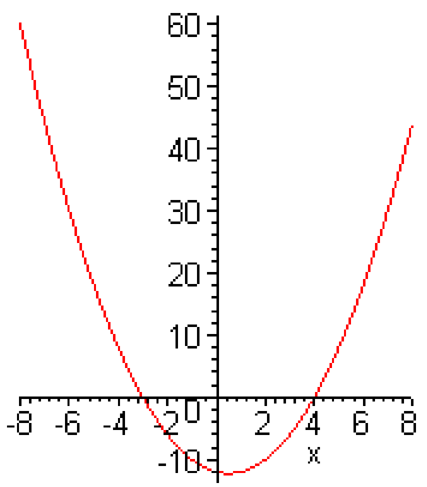
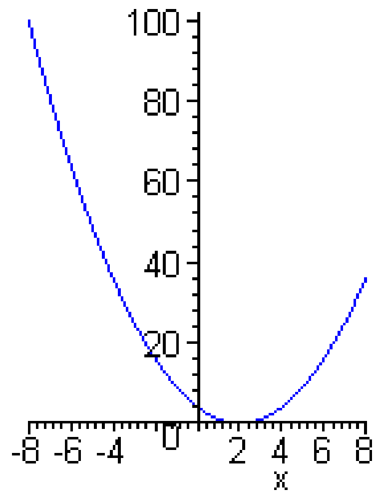
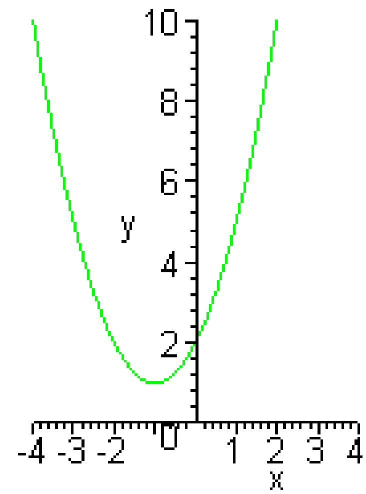
the solution is

$$\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

let us examine the discriminant,

$$b^2 - 4\,a\,c$$

- There are three cases that we need to concern ourselves with, namely: $\mathbf{b^2 - 4ac > 0}$, $\mathbf{b^2 - 4ac = 0}$, and $\mathbf{b^2 - 4ac < 0}$. Here we will graph examples of each of these cases.

Case 1: $\mathbf{b^2 - 4ac > 0}$	**Case 2:** $\mathbf{b^2 - 4ac = 0}$	**Case 3:** $\mathbf{b^2 - 4ac < 0}$
$y = x^2 - x - 12$	$y = x^2 - 4x + 4$	$y = x^2 + 2x + 2$
		

For the case when the discriminant is greater than zero, the parabola crosses the x-axis in two places, indicating two real roots when y = 0. For the case when the discriminant is equal to zero, the parabola intersects the x-axis in exactly one place, indicating one real root. When the discriminant is less than zero, the parabola does not intersect the x-axis at all, indicating no real solutions when y = 0.

Next week, all about **Fibonacci numbers** and how they show up in nature. (I used SnagIt screen capture software to capture these screen pieces, to crop them, to border them, and add the arrows).